

If $f(x) = (\cot x)^{\tan^{-1} x}$, find $f'(x)$.

SCORE: ____ / 25 PTS

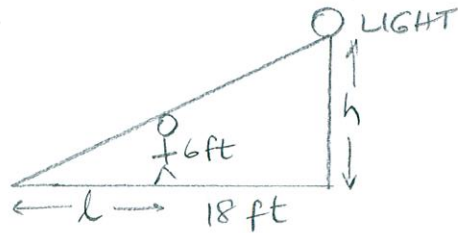
$$\ln f(x) = \tan^{-1} x \ln \cot x$$

$$\frac{1}{f(x)} f'(x) = \frac{1}{1+x^2} \ln \cot x + \tan^{-1} x \cdot \frac{1}{\cot x} (-\csc^2 x)$$

$$f'(x) = (\cot x)^{\tan^{-1} x} \left(\frac{\ln \cot x}{1+x^2} - \frac{\tan^{-1} x \csc^2 x}{\cot x} \right)$$

BJ, who is 6 ft tall, is standing 18 ft away from a floodlight sitting on a stage. If the floodlight is being pulled upward at 2 ft per second, how quickly is the length of BJ's shadow changing when the floodlight is 15 ft above the stage? SCORE: ____ / 25 PTS

NOTE: Give the units of your final answer. State clearly whether BJ's shadow is growing or shrinking.



$$\frac{dh}{dt} = 2 \text{ ft/sec}$$

$$\text{WANT } \left. \frac{dl}{dt} \right|_{h=15 \text{ ft}}$$

$$\frac{l}{6 \text{ ft}} = \frac{l+18 \text{ ft}}{h}$$

$$hl = 6l \text{ ft} + 108 \text{ ft}^2 \longrightarrow h = 15 \text{ ft}$$

$$l \frac{dh}{dt} + h \frac{dl}{dt} = 6 \text{ ft} \frac{dl}{dt}$$

$$(12 \text{ ft})(2 \frac{\text{ft}}{\text{sec}}) + (15 \text{ ft}) \frac{dl}{dt} = 6 \text{ ft} \frac{dl}{dt}$$

$$24 \frac{\text{ft}^2}{\text{sec}} = -9 \text{ ft} \frac{dl}{dt}$$

$$\frac{dl}{dt} = -\frac{8}{3} \frac{\text{ft}}{\text{sec}}$$

BJ'S SHADOW IS SHRINKING

$$15l \text{ ft} = 6l \text{ ft} + 108 \text{ ft}^2$$

$$9l \text{ ft} = 108 \text{ ft}^2$$

$$l = 12 \text{ ft}$$

Let $f(x) = \frac{e^{3-3x}}{5^x - 1}$.

SCORE: ____ / 25 PTS

[a] If x changes from 1 to 1.25, find dy .

$$\frac{dy}{dx} = \frac{-3e^{3-3x}(5^x - 1) - e^{3-3x}(5^x \ln 5)}{(5^x - 1)^2}$$

$$\left. \frac{dy}{dx} \right|_{x=1} = \frac{-3e^0(5^1 - 1) - e^0(5^1 \ln 5)}{(5^1 - 1)^2} = \frac{-12 - 5 \ln 5}{16}$$

$$dy = \frac{-12 - 5 \ln 5}{16} (1.25 - 1) = \frac{-12 - 5 \ln 5}{64}$$

[b] Approximate $f(1.25)$.

$$y(1) + dy = \frac{e^0}{5^1 - 1} + \frac{-12 - 5 \ln 5}{64}$$

$$= \frac{1}{4} + \frac{-12 - 5 \ln 5}{64}$$

$$= \frac{4 - 5 \ln 5}{64}$$

Find the equation of the normal line to the curve $\cos^{-1}(5x + \frac{y^2}{2}) = \frac{2\pi}{y}$ at $(-1, 3)$.

SCORE: ____ / 25 PTS

$$5x + \frac{y^2}{2} = \cos \frac{2\pi}{y}$$

$$5 + y \frac{dy}{dx} = (-\sin \frac{2\pi}{y}) (-\frac{2\pi}{y^2} \frac{dy}{dx})$$

$$5 + 3 \frac{dy}{dx} \Big|_{(-1,3)} = (-\sin \frac{2\pi}{3}) (-\frac{2\pi}{9} \frac{dy}{dx} \Big|_{(-1,3)})$$

$$5 + 3 \frac{dy}{dx} \Big|_{(-1,3)} = -\frac{\sqrt{3}}{2} \cdot -\frac{2\pi}{9} \frac{dy}{dx} \Big|_{(-1,3)}$$

$$5 = (\frac{\sqrt{3}\pi}{9} - 3) \frac{dy}{dx} \Big|_{(-1,3)}$$

$$\frac{dy}{dx} \Big|_{(-1,3)} = \frac{5}{\frac{\sqrt{3}\pi}{9} - 3}$$

$$y - 3 = \frac{3 - \frac{\sqrt{3}\pi}{9}}{5} (x + 1)$$

$$y - 3 = \frac{27 - \pi\sqrt{3}}{45} (x + 1)$$

If $f(x) = \frac{8x^2 - 12x}{\sqrt[4]{x}}$, find $\frac{d^3 y}{dx^3}$.

SCORE: ____ / 15 PTS

$$f(x) = 8x^{\frac{7}{4}} - 12x^{\frac{3}{4}}$$

$$\frac{dy}{dx} = 14x^{\frac{3}{4}} - 9x^{-\frac{1}{4}}$$

$$\frac{d^2 y}{dx^2} = \frac{21}{2}x^{-\frac{1}{4}} + \frac{9}{4}x^{-\frac{5}{4}}$$

$$\frac{d^3 y}{dx^3} = -\frac{21}{8}x^{-\frac{5}{4}} - \frac{45}{16}x^{-\frac{9}{4}}$$

Using the definition of the derivative, prove the derivative of $\cos x$.

SCORE: ____ / 15 PTS

NOTE: You may use the values of the two special limits proved in class without reproving them, if you state the values of those limits.

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} &= \lim_{h \rightarrow 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h} \\&= \lim_{h \rightarrow 0} \left((\cos x) \frac{\cosh - 1}{h} - (\sin x) \frac{\sinh}{h} \right) \\&= \lim_{h \rightarrow 0} \cos x \lim_{h \rightarrow 0} \frac{\cosh - 1}{h} - \lim_{h \rightarrow 0} \sin x \lim_{h \rightarrow 0} \frac{\sinh}{h} \\&= (\cos x)(0) - (\sin x)(1) = -\sin x\end{aligned}$$

MULTIPLE CHOICE. Circle the correct answer. Show work to prove your answer is correct.

SCORE: ____ / 20 PTS

Which of the following curves is orthogonal to the curve $y = \ln(\sec x - \tan x)$?

[a] $y = \sin x$

[b] $y = -\sin x$

[c] $y = \cos x$

[d] $y = -\cos x$

[e] $y = \cos x + \cot x$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\sec x - \tan x} (\sec x \tan x - \sec^2 x) \\ &= \frac{-\sec x (\sec x - \tan x)}{\sec x - \tan x}\end{aligned}$$

$$\text{OTHER } \frac{dy}{dx} = -\frac{1}{-\sec x} = \cos x$$